This approach to the solution of IHP, when the boundary conditions are specified with errors, should be applied, ultimately, only when the distance between points b and d is sufficiently small and the errors in specifying $g(\tau)$ cannot be neglected.

Note, in conclusion, that the methodology of iterative regularization of incorrect problems developed in the last ten years has a broad spectrum of applications in the diagnostics and identification of heat- and mass-transfer processes. This methodology may also have useful applications in solving many inverse problems of mathematical physics arising in other fields of scientific research and engineering application.

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SURVEYS

SOME PROBLEMS OF MASS EXHANGE IN MAGNETIC SUSPENSIONS AND COLLOIDS

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Recently, the attention of specialists has been drawn to different aspects of the interaction of a magnetic field with dispersive magnetized media. A new direction in technology has been developed successfully, the magnetic separation of dispersive materials. Studies have been conducted on the extraction of magnetic components from a nonmagnetic liquid [1-3] and weakly magnetic materials from a magnetic liquid [4-6]. High-gradient magnetic separation of weakly magnetic microparticles has also been intensively studied [7]. Finally, nanoparticles of magnetic liquids are separated magnetically [8]. Magnetophoretic transport of colloidal particles influences the stability of magnetic liquids as well as the working capacity of many technical devices using magnetic liquids (magnetoliquid seals, vibration dampers, printing units with magnetic liquids, etc. [9]). Below we give a concise review of studies on the mass transport of particles in magnetic colloids and suspensions, conducted at the Institute of Physics of the Academy of Sciences of the Latvian SSR.

1. Magnetophoresis of Particles in a Viscous Liquid. The physical basis for the mass transport of particles in a magnetic field is the magnetophoretic force

$$\mathbf{F}_m = V\mu_0 (\mathbf{M}\nabla)\,\mathbf{H}, \tag{1}$$

which is determined by the magnetization of the particle M = M(H) and its volume V. Calculation of the force (1) in the general case of nonequilibrium magnetization is a complicated problem. The quasistationary case $M = \kappa H$ is the easiest one to solve. For this case

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$$F_m = \oint_S \left[H(B\cdot n) - \left(\frac{1}{2}\mu_0 H^2 + p\right) n \right] dS, \quad (2)$$

the induction B and the intensity of the magnetic field are determined by the equations divB = 0 and rotH = 0. Asymmetry of the hydrostatic and magnetic pressure p in (2) has an effect only if the surrounding liquid possesses magnetic properties. A particular example of an ellipsoid of revolution for a given gradient of an external magnetic field in [8] shows that the force F_m acting on a particle with magnetic permeability μ_i = const in the liquid with μ_a = const is of the form

$$F_m = V\mu_0 (M_i - M_a) \nabla H,$$

where M_i and M_a are magnetizations of the liquid and of the body at the intensity of the field inside of the ellipsoid. For other configurations of the particle, when the magnetic field inside it is heterogeneous, the expression for the force F_m is not so simple. However, the general conclusion is preserved: in Eq. (1) the demagnetization factor of the body should be considered.

The rate of magnetophoretic transport of the particle v_m is determined by the balance between the magnetic force and the force of hydrodynamic resistance F_v. In the region Re ≲ 1, the latter is determined by solving the Stokes problem. The magnetophoretic force (1), especially for high-gradient magnetic separation, is strongly heterogeneous; therefore, the hydrodynamic problem should be considered with allowance for a nonstationary term of the equation of motion. Only for nanoparticles and weakly magnetic microparticles, the force F_v can always be determined from the stationary Stokes solution [10]. For spherical particles in the quasistationary approximation, in particular, we have

$$v_m = \frac{9}{2}\frac{\mu_0 a^2}{\eta}(M_i - M_a)\nabla H. \quad (3)$$

We note that an analogous result can be obtained by means of a pseudohomogeneous description of mass transport in dispersive media. As is known [8], magnetodiffusion of paramagnetic ions or molecules in solutions is determined by the relation

$$j_m = D_i \frac{\mu_0(\overline{M}_i - \overline{M}_a)}{c_i \mu_i^c}\rho_i, \quad (4)$$

where D_i is the coefficient of mutual diffusion in a binary mixture, and $\overline{M}_i$ and $\overline{M}_a$ are specific magnetizations of an inclusion and the surrounding liquid. Since for the independently moving components $c_i\mu_i^c \approx kT/m_i$ and $D_i \approx kT/f_i$ (f_i is the coefficient of hydraulic resistance), for the rate of the relative motion of the inclusion in the liquid $v_m = j_m/\rho_i$ in the heterogeneous field, we obtain $v_m = \mu_0(\overline{M}_i - \overline{M}_a)\nabla Hm_i/f_i$. For spherical particles in the Stokes approximation $f_i = 6\pi\eta a$ this dependence coincides with (3). Thus, in principle, there is no substantial difference among the processes of magnetic transfer in suspensions, colloidal solutions, and molecular liquids. The difference is only that in colloids with nanoparticles and in solutions, heat diffusion of the particles should be considered concurrently with magnetic transfer in relation (4).

If the temperature in the magnetic suspension or colloidal solution is not constant, magnetophoresis of particles may be observed even in a homogeneous magnetic field H_0. A theoretical description of this phenomenon calls for a simultaneous solution of the equation of motion, temperature, and magnetic field. If the coefficients of heat conduction of the particles and of the surrounding medium are equal, then for small velocities of the particles, when the Péclet number is small, the temperature perturbations near the particle may be neglected. In this case the local distribution of the magnetic field near the particle is determined from the solution of the equation for the scalar magnetic potential Ψ in the form

$$\Delta\Psi = -\frac{1}{\mu}\frac{\partial\mu}{\partial T}(\nabla T\cdot\nabla\Psi), \quad (5)$$

the hydrodynamic resistance in the Stokes approximation is found from the solution of the equation of vorticity taking account of the magnetic term

$$\Delta\Omega = -\frac{\mu_0}{2\eta}[\nabla\mu \times \nabla H^2], \quad (6)$$

and the thermomagnetophoretic force is calculated from Eq. (2).

From the analysis of these equations it follows that the direction of thermomagnetophoretic transfer depends on the mutual orientation of the magnetic field and the temperature gradient. In the case $H \parallel \nabla T$, particles move in the direction of the temperature gradient with the velocity [11]

$$v_{tm} = \frac{29}{60} \frac{\mu_0 H_0^2 a^2}{\mu\eta} \left| \frac{\partial \mu}{\partial T} \right| \nabla T. \tag{7}$$

If the magnetic field is normal to the temperature gradient, the velocity of phoresis is also directed along ∇T, but in all other cases, particle motion is possible both in the direction of the field as well as in the direction ∇T. In [12], the case is considered when the coefficients of thermal conductivity of a particle and of the liquid are different, and a local distortion of the temperature field occurs near the particle. In this case, Eq. (7) remains valid; however, the numerical coefficient in it depends on the value $K_\lambda = (\lambda_i - \lambda_a)/(\lambda_1 + 2\lambda_a)$. Disregarding the influence of temperature on the distribution of the magnetic field in Eq. (2), it is shown in [12] that the velocity of thermophoresis of thermally nonconductive particles is much higher than the velocity of ideally conducting particles. In greater detail, taking into consideration the effect of temperature on the distribution of the magnetic field in calculating the force F_m, this problem is treated in [13].

In [14], an attempt has been made to investigate experimentally thermophoresis in magnetic colloids. In order to increase the sensitivity of measurements, an experiment was staged in a thermodiffusive column 0.5 mm in width and 70 mm in height. Thermophoresis of colloidal particles was not observed when there was no magnetic field; in a homogeneous field H_0 = 2.3 kA/m at $\nabla T = 4 \cdot 10^4$ deg/m in the case $H \parallel \nabla T$, it has been found that particles move along the direction of heat flow with a velocity of the order of 50 nm/sec. The result obtained is in qualitative agreement with the result of theoretical calculation.

2. Magnetophoresis of Cells and Directed Transport. The measurements of the magnetophoretic mobility of microparticles can be used to determine their magnetic susceptibility. Such a formulation of the problem requires that it be possible to determine exactly the value of the gradient of the magnetic field and hydrodynamic resistance while the microparticle moves in the viscous liquid. These questions are considered in detail in [15] as applied to the problem of magnetic properties of blood cells. The gradient of the magnetic field, high enough to provide for the measurement of the magnetophoretic velocity of weakly magnetic particles, is created by placing a thin wire (30-50 μm in diameter) from a soft magnetic material in the transverse homogeneous magnetic field H_0. If H_0 is less than the magnetization of saturation of the material of the wire, the distribution of the field around a cylinder can be found through calculations, so that the value $K_\mu = (\mu_i - \mu_a)/(\mu_i + \mu_a)$ in the expression for a scalar potential of perturbation of the magnetic field by the cylinder $\Psi = H_0 K_\mu (R/r)\cos\theta$ (θ is the angle between H_0 and the radius vector r) can be treated as independent of the field: $K_\mu = 1$. Some problems arise when determining the hydrodynamic resistance of microobjects, the nonsphericity of particles, their terminal velocity during their motion, and the thermal motion. If measurements are conducted close to the surface of the cylinder, then, when determining the hydrodynamic resistance, the hydrodynamic interaction of the particle with the wall should also be considered [16].

The measurements of magnetic properties of microobjects by determining their magnetophoretic mobility are of great interest in modern biology and medicine, mainly in connection with the development of new methods in medical diagnostics and magnetic transportation of medical compounds. The magnetophoretic method makes it possible to obtain not only the average value of the magnetic susceptibility of the cell, but also the curve of the magnetic histogram and the corresponding statistical characteristics. Correlations between the magnetic characteristics of the biological cells and their biochemical and functional properties appear only for a statistically large amount of data. For an operative analysis of a large array of data, an automated factor for information acquisition and processing was developed [17]. With the use of this apparatus, measurements of the magnetic properties of natural and artificial cells were performed.

The difference in the diamagnetic susceptibilities of particular subpopulations of lymphocytes and the dependence of κ on their functional states were discovered. The heterogeneity of the lymphocytes in magnetic properties, in particular, allows one to divide B-lymphocytes into subclasses in a high-gradient magnetic filter [18].

As is known, modifications of the hemoglobin differ strongly in magnetic properties. Depending on the degree of blood oxygenation, hemoglobin can have paramagnetic or diamagnetic susceptibility. Since hemoglobin is one of the main ingredients of erythrocytes, these differences have a noticeable effect on the magnetic susceptibility of erythrocytes as a whole. In [16], magnetophoretic studies were conducted for human erythrocytes, containing hemoglobin in the form of oxy-, carboxy-, deoxy-, and methemoglobin. It turned out that erythrocytes with oxyhemoglobin ($\kappa = -9.17\cdot10^{-6}$) and carboxyhemoglobin ($\kappa = -9.20\cdot10^{-6}$) have greater diamagnetic properties than erythrocytes containing deoxyhemoglobin ($\kappa \equiv -6.65\cdot10^{-6}$) and methemoglobin ($\kappa \equiv -7.18\cdot10^{-6}$). Therefore, the magnetophoretic mobility of erythrocytes in blood or in a physiological solution $v_m/\nabla H$ can be both negative (with oxy- and carboxyhemoglobin) and positive (with deoxy- and methemoglobin). This allows one to conduct a high-gradient magnetic separation of whole blood, for example, to separate erythrocytes infected by the parasite of malaria, which transforms hemoglobin into the irreversible form of methemoglobin [19]. Measurements of the magnetophoretic mobility of erythrocytes can also be used for diagnostics to determine functional or physiological peculiarities of blood.

Magnetophoresis of cells is also of interest for realization of the magnetic transport of medical compounds. Experiments on localization of magnetic nanoparticles in tissues (with mongrel white rats) and in the vessels of the brain (with rabbits) have shown [20] that comparatively small gradients of the magnetic field of the order of 20-50 T/m are sufficient to retain nanoparticles in the organism both for hypodermic and intravenous injection of the compound. The most promising solution for the problem of directed transport is the use of magnetic microcarriers. These are artificial cells containing a viscous material, magnetic nanoparticles, and a medical compound. Magnetic polyelectrolytic microspheres with rubomicin [21] and curarelike compounds [22] are obtained. The average diameter of cells is about 2 μm, their magnetic susceptibilities vary in the interval from $1.7\cdot10^{-2}$ to $7.7\cdot10^{-2}$ [23]. The achieved values of κ make it possible to localize microspheres in blood flows with velocity of 10^{-3}-10^{-2} m/sec for the gradients of the magnetic field accessible for a biological experiment. Magnetic localization of microspheres with rubomicin in the limbs of mice with a tumor has provided for a considerable inhibition of the growth of the tumor and prolongation of the life of an animal as compared with the standard treatment by this compound without localization [21]. The effect of magnetic localization of the curarelike compounds has been discovered. In particular, in intravenous injection of microcapsules, containing diadonii, more pronounced suppression of the neuromuscular transmission was observed in the limb of a cat which was placed in the magnetic field [22]. The natural cells, for example, colorless erythrocytes, can also be used as microcarriers. For this purpose, magnetic nanoparticles are introduced in their membranes or inside of the cells, by virtue of which the magnetic susceptibilities of the cells can be increased up to the order of 10^{-3} [16]. This is wholly sufficient for the magnetophoretic confinement of erythrocytes in the blood flow.

3. Mass Exchange in Colloids and Suspensions. An analysis, performed by the methods of thermodynamics of irreversible processes, demonstrates that the mass flow of a paramagnetic component in a binary solution taking into consideration magnetodiffusion is described by the dependence

$$\mathbf{j}_i = -D_i\nabla\rho_i + \frac{D_i m_i}{kT}\left[-(\bar{V}_i - \bar{V})\nabla p_1 - (\bar{V}_i - \bar{V})\rho\mathbf{g} + \mu_0(\bar{M}_i - \bar{M})\nabla H\right].$$

Here $\bar{V}_i$ and $\bar{V}$ are specific volumes, and $\bar{M}_i$ and $\bar{M}$ are specific magnetizations of a magnetic component and of the environment. In order to take into account the gravitational sedimentation in the term for the barodiffusion singly, the gravitational term $\nabla p = \nabla p_1 + \rho g$ is isolated from the pressure gradient. As mentioned above, an analogous relation is also true for the magnetic colloidal solution of the Brownian particles for which $D_i = kT/f_i$.

Mass exchange for magnetic sedimentation is determined by a solution of the simultaneous equations of conservation of mass, motion, and continuity, which with no consideration for negligibly small barodiffusion under the action of ∇p_1 and with neglect of the terms pertaining to the intermutual motion of the components of the mixture (approximation of the infinite dilution) can be written in the following dimensionless form [24]:

$$\frac{dc'}{d\tau} = \Delta c' + \nabla\left[A_g\frac{\mathbf{g}}{g} - A_m(\bar{M}_i' - \bar{M}')\nabla H'\right]c', \tag{8}$$

$$\frac{1}{\mathrm{Sc}}\frac{d\mathbf{v}'}{d\tau} = -\nabla p_1' + \Delta\mathbf{v}' - \mathrm{Ra}_c\frac{\mathbf{g}}{g}c' + \mathrm{Rm}_c(M_i'\nabla H')c', \tag{9}$$

$$\nabla\cdot\mathbf{v}' = 0. \tag{10}$$

Here $v' = vL/\nu$, $\tau = D_i t/L^2$, $p_1' = p_1 L^2/\eta D_i$, $c' = c/c_0$, $H' = H/H_0$, $\bar{M}_i' = \bar{M}_i'/\bar{M}_0$, $Ra_c = (\bar{V}_i - V)\cdot gL^3\rho_0 c_0/\nu D_i$ and $Rm_c = \mu_0 \bar{M}_0 H_0 L^2 c_0/\nu D_i$ are Rayleigh gravitational and magnetic criteria, $Sc = \nu/D_i$ is the Schmidt number, $A_g = gL(\bar{V}_i - \bar{V})m_i\rho_0/kT$ and $A_m = \mu_0 m_i \bar{M}_0 H_0/kT$ are dimensionless parameters of the gravitational and magnetic sedimentations.

Numerical estimations show that in magnetic colloids and suspensions, the magnetic terms in Eqs. (8) and (9) are substantially larger than gravitational ones [24]. An essential problem is the relation between the magnetophoretic velocity and thermal velocity of mass transport for colloidal particles in a heterogeneous field, which is considered in the boundary-layer approximation in [10]. An analytical dependence of the Sherwood number on the magnetic sedimentation A_m is found for different values of the diffusion criterion of Biot, and an increase in Bi is shown to accelerate the approach of the dimensionless coefficient of mass exchange to an asymptotic dependence which corresponds to pure convective mass exchange on the surface, the latter being equivalent to a non-diffusional approximation. The obtained solution gives an infinite increase in concentration on the surface for negative values of A_m (a magnetophoretic force is directed toward the mass-exchanging surface), which is physically meaningless. In [25], the nonlinear term with c^2 in Eq. (8) is taken into consideration when analyzing magnetodiffusion (due to the long range of the magnetic forces, the value $\bar{M}$ is in proportion with concentration). The solution, obtained by the integral method, shows that the concentration on the wall and in a flat boundary layer approaches its asymptotic value, depending on c_0. Increase in c_0 leads to the reduction in the magnetophoretic effect.

Neglect of Brownian diffusion for intensive magnetophoresis makes it possible to alleviate sufficiently the analysis of problems of magnetic separation. In this case, a somewhat different system of dimensionless parameters should be used instead of (8) and (9), since the equations of motion and mass conservation should be converted into a dimensionless form using the time of magnetophoresis $f_i L^2/\mu_0 \bar{M}_0 H_0 m_i$ rather than the time of diffusion transfer L^2/D_i. These equations represent a theoretical basis for research in the area of high-gradient magnetic separation, a problem which has recently attracted the wide attention of specialists. The majority of authors operate with the trajectories of particles rather than concentrations [7]. The approach through concentrations is, however, preferable (in analyzing both the problems of high-gradient separation and directed transport) since it allows one, first, to determine the local distribution of concentrations and flows of microparticles in addition to trajectories [26] and, secondly, to discover a number of new properties of mass transfer caused by the formation of concentration fronts and heterogeneities due to the heterogeneity in the force field F_m, the curvature of the surfaces of localization, and the heterogeneity in the velocity profile [27, 28]. Fairly simple analytical solutions of problems of mass exchange in a nondiffusional approach serve as a good test for convergence and asymptotical behavior of numerical solutions of magnetodiffusion problems [29].

Attention should be drawn to the basic importance of the question of boundary conditions for problems of magnetodiffusion in magnetic liquids. The value of the Biot diffusion criterion affects substantially the intensity of mass exchange for both filtering elements of high-gradient separators and magnetoliquid devices. In theoretical studies of these problems, the condition of the closed surface $j_{iw} = 0$ is usually given. This condition, unless otherwise specified, is applicable only to ideally stable magnetic liquids. However, experimental investigations show that even in this case, especially when the gradient of the magnetic field is high and directed toward the surface, a portion of the colloidal particles settle irreversibly onto mass exchange surfaces [8]. There are a number of studies [30] in which the kinetics of magnetic flocculation is considered in colloidal dispersed solutions in a heterogeneous field. However, the features of the interaction of particles with a massive surface are unknown; therefore, an experimental determination of this interaction is of great interest. In our investigations, such an investigation was attempted by studying mass exchange on the surface of a transversely magnetized microcylinder. Numerical investigations show [29] that the concentration of colloidal particles near the surface in the regions of their accumulation is very sensitive to the value of the Biot criterion. Experiments using the method of double exposure holographic interferometry under a microscope show [25, 31] that the pattern of distribution of the concentration of particles near the cylinder depends to a high degree on the degree of aggregational stability of the colloid. The dilution of water-based magnetic liquids entails, as a rule, a change in the pH of the medium and deterioration in stability of the colloid. In this case one can observe the irreversible sedimentation of particles on the surface of the cylinder in the regions of the maximal magnetic field. In kerosene magnetic liquids, when the measures necessary for the conservation of the absorption layers of surface-active materials on particles are followed carefully as the liquid is diluted, the capture of nanoparticles of some 5 nm in diameter by the surface of the cylinder, especially for small magnetic-field intensities, is not practically observed. The quantitative analysis of the results and the determination of the value of Bi, however, are difficult since the exact measurement of the concentration on the surface of the cylinder is impossible due to the strong absorption of light in the regions of accumulation of particles and due to the thin structure of interference bands. We direct our attention to the qualitative difference between experimental and theoretical profiles of concentration of particles far from the cylinder, calculated in the approximation of nonstationary magnetodiffusion. This is caused by magnetic convection.

4. Magnetodiffusive Convection. As is known [8], the condition for the development of convection without a threshold in a magnetic liquid at $M = \kappa H$ can be written in the form

$$\nabla\rho \times \mathbf{g} + \mu_0 \nabla M \times \nabla H \neq 0.$$

The spatial heterogeneity in the density ρ and magnetization M is usually assumed to arise only in nonisothermal conditions. However, ρ and M depend not only on the temperature but also on the concentration of the magnetic particles. Since magnetophoresis in a heterogeneous magnetic field results in a nonuniform distribution of particles, magnetic convection arises also under isothermal conditions [24, 32]. The latter is analogous in many respects to free convection, appearing with gravitational stratification of particles. The condition for the formation of magnetic convection without a threshold in the isothermal liquid when there is no gravitation is of the form [8, 24]

$$\nabla c \times \mu_0 \frac{\partial M}{\partial c} \nabla H \neq 0. \tag{11}$$

Since the magnetic force in the equation of motion (9) and the force of magnetophoretic transfer in the equation of diffusion (8) depend on the same vector factor $M\nabla H$, condition (11) holds obviously only when the solid boundaries between which the magnetic liquid is confined are not orthogonal to the vector $M\nabla H$. Such a situation is realized in particular in the experiment on mass exchange on a transversely magnetized cylinder [31]. Numerical modeling of the magnetodiffusive convection for this geometry has been conducted in [24, 32]. Experimental profiles of concentrations [31, 33] are in good agreement with the results of numerical calculation. This can be considered as an experimental corroboration of the existence of magnetodiffusive convection. Since in magnetic liquids, the Rayleigh magnetic number for concentration is a few orders higher than its thermal analog, magnetodiffusive convection is substantially more intensive than thermomagnetic convection. In [33], on the basis of the solution of the boundary-layer problem, it has been shown that for field gradients characteristic of high-gradient magnetic filters, the velocity of magnetic convection in the neighborhood of filtering elements, even for typical size of the order of 0.1 mm, can achieve several centimeters per second.

It has been found that magnetodiffusive convection can arise also when the isoconcentration lines are normal to the magnetic force $\mu_0 M\nabla H$. For example, in a strongly heterogeneous field, when the magnetophoretic force increases in the direction of motion of particles, a concentration gradient that is antiparallel to the vector $M\nabla H$ can be created. A problem arises on the convective instability, similar to one which describes the emergence of Benard convection in the gravitational field. As an example of this magnetoconvective instability, in [34] the problem of nonstationary diffusion is considered in the heterogeneous magnetic field of a cylindrical conductor with axial electric current I when $H = I/2\pi r$ and the phoretic force in the region of paramagnetic magnetization of the colloid in the direction of motion of particles increases according to the law $F_m \sim r^{-3}$. In a definite time interval there is a zone of inverse concentration gradient in the neighborhood of the cylinder. On the basis of numerical investigation of the evolution of random harmonic perturbations, it is shown in [34] that there is a critical value of the magnetic Rayleigh number depending on the parameter of magnetodiffusion, time, and the diameter of the cylinder, above which random perturbations start increasing. With an increase in time, upon having achieved a stationary distribution of concentration, all perturbations, of course, are quenched.

The intrinsic field of the magnetic liquid also produces a definite effect on the convective stability of the liquid under magnetic stratification of particles. In [35], the deterioration in stability of the diffusion front of particles is considered under gravitationally stationary stratification of the magnetic liquid, caused by the homogeneous magnetic field transverse with respect to the diffusion front. The cause for generating a cellular convection in this case is the initiation of the heterogeneous intrinsic magnetic field of the ferroliquid, the intensity of which is directed to the lower concentrations. In [36], the numerical modeling of this problem is realized. On the basis of the numerical solution of the system of equations, describing the development in time of the field plane concentrations as a function of a current, concentration field, and the potential of the intrinsic magnetic field, the rate of growth of the amplitude is determined and the evolution of initial random perturbations is analyzed. The convective motion is shown to result in an increase in the effective width of the diffusion front. The initiation of microconvection in the diffusion front has been discovered experimentally [35].

NOTATION

A_g, A_m, dimensionless parameters of the gravitational and magnetic sedimentation; a, radius of particles; B, magnetic induction; c, volumetric concentration of particles; D_i, diffusion coefficient; F, force; H, magnetic intensity; k, Boltzmann constant; L, characteristic dimension; M, magnetization of a unit volume; $\bar{M}$, specific magnetization; m_i, mass of particles; n, normal to the surface; R, radius of the cylinder; p, pressure; Ra_c and Rm_c, gravitational and magnetic Rayleigh numbers; Sc, Schmidt number; T, temperature; V, volume; $\bar{V}$, specific volume; η, dynamic fluid viscosity; κ, magnetic susceptibility; λ, thermal conductivity; μ, magnetic permeability; μ_0, magnetic constant; ρ, density.

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TEMPERATURE FIELDS AND STRESSES IN BODIES WITH DISCONTINUOUS PARAMETERS

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Bodies with discontinuous parameters are utilized extensively as structural elements in different areas of modern engineering. Among them are thermally sensitive, piecewise-homogeneous and multistage bodies; bodies with piecewise-constant coefficients of heat elimination from their surfaces, with cutouts, holes, gaps, of finite dimensions. The physicomechanical characteristics of multistage structural elements can be described as a single whole for the whole body by using asymmetric unit functions, while bodies with a continuous inhomogeneity and bodies with temperature-dependent properties (thermally sensitive bodies) are approximated by using these functions. The desired functions (the temperature field t, the stress tensor components σ_{ij}, and the displacement vector u_i) for bodies with cutouts, gaps, holes, of finite dimensions can be continued into the domain enclosing the cutout, gap, hole; in an infinite domain in one dimension, respectively. For the bodies under consideration this permits writing integrodifferential (generalized problem) or differential equations (classical problem) of heat conduction and thermoelasticity with discontinuous and singular coefficients. For bodies with discontinuous coefficients of heat elimination, the boundary conditions can be written with discontinuous coefficients. Investigations executed in this scientific direction are generalized in the monographs [1-4] and analyzed in [5]. Here we examine the analysis of further investigations performed in the area of the heat conduction and thermoelasticity of bodies with discontinuous parameters by using the apparatus of generalized functions.

Let us consider an anisotropic inhomogeneous body occupying a domain V and having the temperature t_0 in the undeformed and unstressed state. A system of differential equations is obtained in [6] for the generalized interconnected dynamic problem of the thermoelasticity of an anisotropic inhomogeneous body under the assumption that the relaxation time of the heat flux does not change as a function of the coordinates, which is valid for metals [7]. If the relaxation time $\tau_r(M)$ depends on the coordinates, i.e., the generalized heat-conduction law has the form

$$lq_i = -\lambda_{ij}^t (M) t_{,j} \quad (M \in V,\ i,\ j = 1,\ 2,\ 3), \qquad (1)$$

the heat-conduction equation for an anisotropic inhomogeneous body will then be written for $\dot{t}/_{\tau=0} = 0$ as

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